Pitch class hierarchies in Miles Davis’ »So What«: Reconsidering modal jazz improvisation with computer-based analysis tools
In jazz studies and jazz theory, »modal jazz« is a well-established, but rather vague term. As jazz theorist Mark Levine puts it,

»modal tunes provide much more space for improvising on each chord compared to previous jazz tunes and standards […] Because of this, it was natural for musicians to focus on the scale, or mode, of each chord, rather than on the chord itself.«

However, several authors emphasize that improvising on a certain scale or »mode« does not necessarily entail being restricted to the pitches of only that scale. For example, Barry Kernfeld writes that, »modal improvisation often unfolds in a flexible and unsystematic way that undermines the identity of specific ethnic or ecclesiastical modes […]«. Although some of the church modes (Phrygian, Dorian etc.) are employed in modal jazz composition and improvisation, there does not appear to be any direct relation to medieval or non-Western practices of modality. However, there are links to African American music traditions, such as the blues, which are rooted in a non-functional harmonic organisation of sung melodic patterns accompanied by guitar or piano patterns. Robert Hodson states that modal jazz could be better characterised as a »succession of static yet colourful blocks of music, and this stasis is a result of treating modes as collections of pitches in a way that relaxes the forward-moving tensions associated with goal-oriented tonal music«. Keith Waters sums up the discussion about modal improvisation in jazz research by indicating the different meanings of the term, ranging from improvisation over scales (instead of chords) and forms with a slow harmonic rhythm, often over pedal-points, to non-functional harmonic progressions in post-1960 jazz.

This broad usage of »mode« and »modal« in contemporary jazz, however, tends to dilute the original meaning of these terms. In music theory and ethnomusicology, the terms are applied to »open-ended heterogeneous networks of melodic types« on the one hand, and to »closed systems of music-theoretical categories« on the other. As Lewis Porter points out, jazz theorists and jazz musicians generally use »mode« in this second, »closed-system« sense and define modes as types of scales. »By the mid-18th century«, Powers writes,

»mode« in European languages meant a collection of degrees of a scale (and its aggregate intervallic content) being governed by a single chief degree: a mode was a scale with a tonic, which was the last note of a melody or the root of a final triad. This is the sense in which the major and minor scales, as well as the so-called

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1 Mark Levine, The jazz theory book, Petulama 1995, chapter »modal jazz«.
church modes, are still deemed modes, and it is with this sense that application of the term modes to phenomena and practices in other musical cultures first appeared.8

Moreover, the concept of a modes or scale often implies not only a tonic, but also a pitch class hierarchy, i.e. certain scale degrees are more important than others.

Does this concept of modality hold true in regard to modal jazz improvisation, e.g. to the Miles Davis Sextet’s seminal recording Kind of Blue, one of the first examples of modal improvisation in jazz history? The career of the terms modes and modality in the context of jazz improvisation probably began with comments made by trumpeter Miles Davis and pianist Bill Evans concerning the use of scales in their music around 1958/59. In an interview with journalist Nat Hentoff in 1958, Davis said:

»I think a movement in jazz is beginning away from the conventional string of chords, and a return to emphasis on melodic rather than harmonic variation. There will be fewer chords but infinite possibilities as to what to do with them.«10

Davis emphasises his attempts to escape functional harmony (changes), while benefiting from the opportunities offered by melodically inventive improvisation:

»All chords are relative to scales and certain chords make certain scales. When you go on this way you can go on forever. You don’t have to worry about [chord] changes and you can do more with the line. It becomes a challenge to see how melodically inventive you are.«11

In the liner notes to the album Kind of Blue, recorded by the Miles Davis Sextet in 1959, pianist Evans comments on the first piece So What: So What is a simple figure based on 16 measures of one scale, 8 of another and 8 more of the first […]12 Neither Evans nor Davis employ the terms modes or modality, but rather the term scales. However, in the liner notes, Evans writes that a few modal changes, in combination with Miles’ free melodic conception, create the mood of another of the Kind of Blue pieces, Flamenco Sketches. Davis and Evans may be referring to jazz composer and theorist George Russell’s early attempt to relate chords to scales; Russell’s Lydian Concept of Tonal Organisation13 was published in a second, more widely distributed edition in 1959.14

In this paper, we examine the selection, frequency, and hierarchy of pitches used in modal jazz improvisation by soloists and accompanying musicians, especially the bass player, who improvises at the ground level. Do the soloists and bass player treat the modes prescribed in the composition simply as a collection of pitches or do they also impose pitch class hierarchies? Are there any remnants of earlier practices of jazz improvisation, e.g. routines of ii-V-cadences, leading and passing tones, or chromaticism in evidence? Do the solo and bass lines outline the harmony in conventional ways or do they adapt other strategies, e.g. changes of tonal centres or chords?

We would like to explore these issues by examining in detail the seminal modal jazz recording So What (1959) by the Miles Davis Sextet. We will start with a brief outline of the composition’s melodic and harmonic framework. Our main contribution lies in a detailed analysis of pitch frequencies and hierarchies

10 Nat Hentoff, »An Afternoon with Miles Davis«, in: Jazz Review 1/2 (December 1958), p. 11.
11 Hentoff, »An Afternoon with Miles Davis«, p. 11.
12 Bill Evans, »Improvisation in Jazz« Liner Notes to Miles Davis’ Kind of Blue, Columbia CL 1355.
14 Waters, The Studio Recordings, p. 42.

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within the improvisations of Davis, John Coltrane and Julian "Cannonball" Adderley, as well as the accompanying walking bass lines played by Paul Chambers, with the aid of computer-based analysis tools developed within the Jazzomat Research Project. Finally, we will compare our findings with two solos by Coltrane over "Impressions" – which is based upon the same harmonic framework as "So What" – as well as with pitch distributions in more conventional jazz improvisation.

»So What« – The Miles Davis Sextet 1959

*Kind of Blue* was recorded during two sessions in March and April 1959 in New York City by the Miles Davis Sextet, consisting of Davis (tp), Coltrane (ts), Adderley (as), Evans (p), Chambers (b) and Jimmy Cobb (dr). In September 1958, Davis had already recorded "Milestones", where no chords were prescribed for improvisation, but instead, a succession of four scales or modes. Half a year later, with the recording *Kind of Blue*, he definitively established his new strategies for harmonic and improvisational organization, in particular with the two modal compositions "So What" and "Flamenco Sketches". The musicians around Davis entered the studio without knowing the tunes. They performed the takes almost off-the-cuff from sketches, with very little in the way of preparation or instructions from Davis. However, the musicians might have been told to expect something similar to "Milestones" before the recording dates, so there could conceivably have been some mental preparation involved.

In "So What", credited to Davis, the bass takes the main melody – a practice that was very uncommon back then and still is today. The overall form is AABA (or AAA'A), with each form part lasting 8 bars. The B section consists of an identical transposition of the A section one semitone upwards. The bass melody consists of four short phrases that start with seven eighths (upbeat), followed by a long tone on the beat. The motivic structure is \(a a' a''\), whereby \(a\) ends on the tonic, and \(a'\) and \(a''\) end on the fifth. The \(a'\) motif may imply a ii-V movement due to its ending on A. The bass motifs are always answered by two chords one whole tone apart (\(E - A - D - G - B\) followed by \(D - G - C - F - A\)), which are played by the piano and doubled by the horns. This widely spaced voicing is based on a stack of three fourths and a major third and is tonally rather ambiguous. The chords could be interpreted both as m\(^{11}\) chords, viz. Em\(^{11}\) and Dm\(^{11}\) (A sections), as A\(^{9}\)sus\(^4\)/E – G\(^{9}\)sus\(^4\)/D, but also as the V-I movement A\(^{9}\)sus\(^4\)/E – Dm\(^{11}\). Taken together, the chords already contain the full Dorian scale as their pitch material. The bass motif also clearly exposes the Dorian scale, although it does not employ the minor third. All in all, the theme of the tune clearly sets the "modal scene", but at the same time echoes a functional harmony and cadential movement. In addition, the change to Eb Dorian in the B section can be viewed as a remnant of cadential thinking, since Eb is the tritone substitute of the dominant A. Hence, the B section has a higher tension, demanding a resolution back to the tonic mode.

After the theme, the solos by Davis, Coltrane, Adderley follow, as well as a rather sparse piano solo played by Evans. The solos by Davis, Coltrane, and Adderley are all of equal length (two 32-bar choruses...
with an overall duration of approximately two minutes each) and appear to be based on the modal structure of D and E♭ Dorian, as prescribed by the composition. The different melodic and motivic characteristics of the three solos have already been described by several jazz researchers. Lewis Porter sums up that,

>«[c]h each soloist on «So What» chose a different solution to the challenge it posed. Alto saxophonist Cannonball Adderley implied diatonic progressions over the sustained modes. [...] Opening the improvisations, Davis brilliantly worked with short, tuneful motives [...] Coltrane spontaneously composed a tightly unified solo notable both for the abstract quality of its melodic motives, and for the way he develops each of them.»

In the following, we will focus solely on the usage of pitch classes in the improvisations of Davis, Coltrane and Adderley. It is not our intention to add any new observations or to generate new insights in regard to the motivic and rhythmic strategies in the solos of «So What». However, it should be mentioned that the three musicians did not restrict their modal approach towards modal improvisation to compositions like «So What», but improvised with a similar, scale-orientated approach around conventional harmonic frameworks. As Kerschbaumer states, Davis started improvising with one row of tones over the harmonic changes of an entire composition, e. g. «Autumn Leaves», «Something Else», or «One For Daddy-O» as early as 1958. Therefore, improvising over a harmonic framework that is reduced to a few scales could be seen as an approach that is applicable not only to original modal compositions like «Milestones» or «So What», but also to more conventional jazz compositions.

Method

We examined the pitch class frequencies and hierarchies within the improvisations using the melfeature module of the MeloSpyGUI and transcriptions from the Weimar Jazz Database, both developed within the framework of the Jazzomat Research Project. Using MeloSpyGUI, we extracted pitch sequences, structural markers which indicate form part, metrical position (on/off beat), syncopations, as well as duration classes (classified according to five grades: from very short to very long) for each solo. This information was imported into the statistical software R for further analysis. After this, we examined the frequencies and the hierarchical organisation of pitch classes. The walking bass lines of Chambers that accompany the three solos were automatically extracted with the aid of an audio analysis algorithm and subsequently

Consequently, the transcriptions are sometimes more difficult to read than other available transcriptions, e. g. the transcriptions of Porter, John Coltrane, pp. 163ff., Kerschbaumer, Miles Davis, pp. 206ff., or Johann Kavarza, Julian «Canonball» Adderley (1928–1975). Seine Improvisationstechnik in der Zeit seines Schaffens bei Miles Davis (Jazzforschung 18), Graz 1986, pp. 54–56.

However, not every musician who played with Davis was open to this kind of a scalar or «modal» approach. Bebop saxophonist Sonny Stitt, who played «So What» in 1960 as a member of the Miles Davis group, had a more conventional, triad-orientated approach. As Jimmy Heath, who substituted for Stitt, reported: «Sonny Stitt did play it like it was a D minor chord, and Miles didn’t want it to be like that. He wanted it to be all the white keys so that it could be C, F, it could be all other things happening.» Cited in: Porter, John Coltrane, p. 162.

Kerschbaumer, Miles Davis, pp. 86–89.

For a detailed description of MeloSpyGUI, see http://jazzomat.hfm-weimar.de/documentation.html.

See http://jazzomat.hfm-weimar.de/dbformat/dboverview.html. The solos were transcribed manually by jazz and musicology students in a MIDI-like format with the help of Sonic Visualiser software which allows for a continuous aural comparison between the original audios and the transcribed note layer. The note layer contains pitch, onset, and duration of each tone; another annotation layer contains time points for beats, which were tapped manually by the transcribers, chords taken from Real Books and other sheet music sources, and overall musical form (e. g., AABA-sections, choruses). Additionally, there are separate layers for phrases (i. e., grouping of melody tones) as perceived by the transcribers, and articulation, e. g. pitch glides, growls, or vibrato.

aurally corrected. The extraction was based on the annotated beat track. This resulted in a bass pitch for every played beat, 256 in total for each solo. Most of the time, Chambers plays straight quarter notes with few embellishments and rhythmic variations, so that the transcription is a good approximation of the actual bass line. Transcriptions of all three lines can be found in the Appendix. Pitch content, form and chord context, and the metrical positions of each beat were also imported into R for analysis.

**Davis’, Coltrane’s and Adderley’s solos in »So What«**

Davis plays 221 tones\(^{28}\) during his two choruses, while the two saxophonists perform roughly twice as many: Adderley 445 tones and Coltrane 479. Davis’ solo appears to be far more relaxed and »spacious«, as compared to Coltrane’s and Adderley’s solos, simply due to the far smaller amount of tones he plays.

In order to investigate whether there is a tonal hierarchy employed by the players, we first ranked the pitch classes of each solo with respect to the frequency of their occurrence for each section (A, B) separately, pooled across the soloists, as can be seen in Fig. 1.

![Figure 1](image)

**Figure 1** Pitch classes ranked by frequency per form part, pooled across the three soloists.

Chromatic tones, which are not included in the Dorian scale, constitute about 10% of all pitches in the solos, ranging from 6% for Davis, over 10% for Adderley, to 12.5% for Coltrane. A relatively clear pitch class hierarchy can be seen for both parts. Using k-means clustering for both form sections separately, after determining an optimal cluster number with the Variance Ratio criterion according to Calinski and

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\(^{28}\) The first two tones on the upbeat were excluded from the following analysis due to technical reasons.
Harabasz, we obtained a four cluster solution for the A sections and 3 cluster solutions for the B sections, which are depicted in Table 1.

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<td>I</td>
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Table 1 Pitch class hierarchies for the two form sections.

Note: p is the relative frequency of the pitch class in the corresponding section. Level was determined by k-means clustering, in which the optimal cluster number was determined using the variance ratio criterion according to Calinski and Harabasz. For the A section, the most important pitches are D and A (Level 1 in Tab. 1), followed by E, G, F, and C in Level 2. This is followed by a third group consisting of the pitches B and D⁷/C⁷. Interestingly, the major sixth, B, the discriminating interval between D Aeolian and D Dorian, is used rather rarely. The last group contains the chromatic pitches A⁷, B⁷, E⁷, and G⁷. For the B section in E⁷ Dorian, the hierarchy is

30 Calinski and Harabasz, »A dendrite method for cluster analysis«.
31 While the tones might be better spelled as either X⁷ or X⁹ according to their specific tonal context, only X⁷ is used in the figures.
even more clear-cut. The most frequent pitches are $B_b$ and $E_b$. However, $D_b$ and $A^b$ occur with almost the same frequency and these four pitches form the first level. This could be interpreted as an indication of the soloists partly combining the $E_b$ Dorian mode prescribed by the composition with $D_b$ Ionian, $A^b$ Mixolydian and other related modes. The next cluster (Level 2) contains the remaining pitch classes of the $E_b$ Dorian/$D_b$ Ionian scale: $F$, $C$, and $G$. The last cluster consists of the chromatic pitches $D$, $E$, $A$, $B$, and $G$. This indicates that the soloists maintain a strong feeling of tonality in the sense of pitch class hierarchies.

![Figure 2](image-url) Pitch classes ranked by frequency of occurrence for form parts A and B for the three soloists.

However, there are individual differences between the players. The same ranked pitch frequencies differentiated according to the respective players can be found in Fig. 2. A clear staircase pattern can be seen for all three soloists and for both modes. The most extreme hierarchy can be found in the D Dorian sections of Davis’ solo. The most frequent pitch classes are $D$ and $A$, followed by the remaining pitch classes $G$, $C$, $E$, $F$ from the D Dorian scale. Of the remaining five pitch classes, Davis uses only $D^b/C^#$ and $A^b$, mostly in an approach to the fifth ($A^b\rightarrow A$) or as leading tone to the tonic ($C^#\rightarrow D$). The pitch class hierarchy for Coltrane’s A sections is similar to that of Davis, with the fifth $A$ being the most frequent pitch, followed by the tonic $D$, the third $F$, and the fourth $G$. The next group consists of $E$, $C^#/D^b$, and $B$. The rather high ranking of the leading tone results mostly from the last A section in his first chorus (A3-1) and the first A section in his second chorus (A1-2), where Coltrane varies a single motif containing the leading tone throughout (cf. Fig. 3). The last group again contains the remaining chromatic tones. Compared to Davis’ solo, Coltrane’s pitch class hierarchy follows more of a linear descending course. However, the pitch class hierarchies of Davis and Coltrane in the B sections are both less strongly terraced than those of the A sections. Interestingly, Coltrane prefers $E^b$ and $A^b$, suggesting $A^b$ Mixolydian, whereas Davis seems to lean towards $D^b$ Ionian, with $D^b$ and $A^b$ as his most frequent pitches.
To further illustrate this observation, the pitch classes are depicted with respect to all eight sections in Fig. 3. The picture is not as clear-cut as the pooled distribution might suggest. All three soloists start their first eight bars with only a few pitch classes: Davis with 5, Coltrane with 6, and Adderley with 7. Davis maintains this simplicity for most of his D Dorian parts and uses a wider variety of pitch classes in the B sections.

In contrast to Davis and Coltrane, Adderley’s hierarchy appears to outline the A Aeolian mode – especially in his second chorus with pitch classes E, A, C, D, G in the main group and a transition group of F and B. This might result from playing within the upper structures of an underlying Dm7 chord, and could be seen as one reason why Adderley’s solo appears to be more rooted in bebop style of improvisation than those of the other two soloists. With respect to the B sections, the most frequent pitch classes employed by Adderley are B♭ and D♭, followed by F, C, Gb, Ab and Eb on a nearly equal footing. This again suggests B♭ Aeolian rather than E♭ Dorian. In general, all players use more chromatic pitches in the B sections, with Adderley using the most. Interestingly, Adderley’s first two A sections suggest C Ionian, whereas his last two A sections (A2-2 and A3-2) lean towards A Aeolian, according to the most prominent pitch classes C, D, E and G or A, respectively. The middle A section appears to oscillate between these two possibilities. Adderley’s B sections lean fairly consistently towards B♭ Aeolian, Coltrane’s first B section towards A♭ Mixolydian and his second B section more towards D♭ Ionian, although the most frequent pitch class is Eb. His A sections tend towards D Dorian, except for A3-1 and A1-2, which could be interpreted as D melodic minor due to the presence of the leading tone C♯/Db, and A3-2, which tends towards A Aeolian. Davis also uses mainly D Dorian for his A sections, except for A1-2, which tends towards C Ionian, and A3-2, which tends towards A Aeolian. His B sections tend rather more towards D♭ Ionian, with E♭ Dorian as a close second interpretation.
However, is pitch class hierarchy determined by the frequency distribution of pitch classes alone? Some tones might be perceptually more salient and therefore more important within the pitch class hierarchy than others, due to certain properties, including a longer duration, their structural position, e.g. at the start or end of a phrase, or their metrical weight. In order to examine these issues, we first split the pitch classes into two groups, one with tone events on a beat (1, 2, 3, or 4) or syncopated tones (i.e., tones before a beat with no tone on the following beat), the other with tones between beats that are not syncopations. We conducted a χ²-test between the two groups for each form part separately. The test for the A sections barely reached significance on the 5% level (χ²(11) = 18.99, p = .061). However, after excluding the A3-1 parts played by Coltrane, which employ the leading tone C♯ rather frequently, the test yielded no significance (χ²(11) = 10.83, p = .46), very similar to the test for the B sections (χ²(11) = 10.803, p = .46). Another set of tests for the differences between events on strong beats of 4/4 time (on 1 or 1 and 3) and all other events, were likewise not significant. For tones that start or end a phrase in comparison with all other tones, there was a significant difference (χ²(11) = 46.316, p < .001) for the A sections, but not for the B sections (χ²(11) = 11.901, p = .47). The effect for the A section was mostly due to a higher frequency of the tonic D played at phrase boundaries. Separate tests for each player showed that this holds true for Davis and Coltrane, but not for Adderley. The distribution of pitch classes with respect to the duration of tones yielded no reliable results, since longer events (quarter notes and longer) are considerably rare in comparison to eighths, sixteenths and shorter tones.

On the whole, the analysis of the solos has shown that the A sections are tonally clearly outlined by all three players, and do indeed emphasize D Dorian. In contrast, the B sections are peaks of tension, with a heightened use of chromaticism and less clear and more ambiguous pitch class hierarchies. There is a strong tendency prevalent among the musicians not to use E♭ Dorian, but rather a mixture of modes with a strong tendency towards D♭ Ionian and A♭ Mixolydian. In both sections, the musicians are not restricted by certain pitch class hierarchies but can alter these hierarchies temporarily, as the melodic minor motif in sections A1-3 and A2-1 of Coltrane’s solo shows. Also, playing in modes starting at higher scale degrees of the prescribed Dorian mode (fifth, seventh, etc.), the «modes» of the original mode, is a technique employed by all three players to some extent: Adderley more so, Davis less, although the clearest example of this technique can be found in the first A section of Davis’ second chorus, where he plays the tones of the C major chord for nearly seven bars and then ends with a cadential figure in F major (m. 41). In general, Adderley uses E, A, and C more frequently than the tonic D in the A sections – which indicates that he tends to play in A Aeolian, a fifth above D, rather than D Dorian; this could be interpreted as a parallel to bebop strategies of employing the higher intervals (7th, 9th, 11th etc.) rather than the triad of a given chord. Nevertheless, he starts his solo with an explicit statement of the D Dorian scale.

Bass

It is the full context of the band’s interplay that provides a degree of harmonic stability. As Hodson states, the listener hears Davis’s solo within the musical context provided by the rhythm sections. Therefore it would be necessary to analyse the bass and piano parts in order to fully appreciate the soloist’s improvisation. In particular, Evans’ accompaniment provides colourful and spacious piano voicings of combined

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32 We are well aware that the precondition of independent events for conducting χ² tests is not entirely fulfilled, since the pitches are part of a time series with correlations between consecutive events. However, these correlations have only a short range of one or two events. Therefore, most events can be considered as practically independent, which seems an acceptable precondition for using the χ² tests. The only significant relation was moreover highly significant so that we do not anticipate any problems with false positives here.

33 Hodson, Interaction, improvisation, and interplay in jazz, pp. 16–21.
4ths and 3rds with tones from the scale, moving diatonically through the pitches. 34 Though a complete transcription of Evan's voicings would be insightful, we will focus here on the bass’ playing and its usage of the modes from a horizontal, melodic perspective.

Since the only information for the bass player is the AABA-form with eight measures each and the prescribed Dorian modes for each section, the question is whether and how the formal structure and the harmonic shifts are outlined in a recognizable manner, and how tonal stability or instability is created by the bass lines.

Since the A and B sections share the same mode, a promising approach would be to compare all those sections separately. As can be seen in Fig. 4 and Fig. 5, the bass lines in the A sections during all three solos establish a fairly clear minor mode with D as the tonal centre.

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34 Ibid., p. 19.
Figure 5 Pitch classes of walking bass tones ranked by frequency for all 8 form parts, differentiated according to soloists.

Figure 6 Pitch classes of walking bass tones on the downbeats ranked by frequency for all eight form parts, differentiated according to soloists.
The distribution of bass tones on the structurally important downbeats (Fig. 6) corroborates this observation. In the A sections, most bars start with a D or an A. During Davis’ solo, only two of 48 measures do not begin with a tone of the Dm7 chord (m. 14; m. 45), none during Coltrane’s solo. Furthermore, Chambers frequently exhibits an ascending I-2-3 line in D minor, as well as an ostinato consisting of tones from the D minor triad during the first A sections of Davis’ and Coltrane’s second choruses. In contrast, during Adderley’s solo, Chambers destabilizes the A section more frequently. The first slight destabilization, right at the beginning of Adderley’s solo, uses different tones of the D Dorian mode on beats 1 and 3, starting with an E (m. 1); later in his solo he plays a G (m. 27) and even a Bb (m. 46).

In addition, Chambers supports the tonal centre with frequent implied dominant chords on A, however, most of the time avoiding the leading tone D/C#.35 Though Chambers also avoids clear cadences or turnarounds at the end of each section, he often makes use of ambiguous lines right before and at the end of the B sections, where he blurs the underlying modes by playing mostly chromatic lines. This is in line with frequent anticipations of the following mode by the soloist, which prepare the ensuing shift.

Another important feature of Chambers’ playing are coherent lines that span more than one bar. For instance, he rather frequently plays a falling D Dorian line D–(Bb–)C–B–(Ab–)A–G–F–E (A) over two bars (e.g. during Davis’ solo m. 9–10), with a chromatic passing tone, so that the first measure emphasises the tonic D and the second the fifth A. In general, Chambers has a tendency to use longer lines with consistent directions (ascending/descending). Using a baseline of walking bass lines extracted from 217 solos from the Weimar Jazz Database in a swing feel, we compared the mean run lengths of ascending, descending, and constant intervals from this set with those of Chambers’ run lengths. The difference is highly significant (Kruskal Wallis test, $\chi^2(1) = 149.61, p < .00001$), with a mean run length of 3.87 for Chambers as compared to a mean run length of 1.42 for all other bass lines. The standard deviation of Chambers’ run lengths is 3.75, due to very long lines with up to 21 intervals in the same direction (during Adderley’s solo in m. 56–61, where Chambers moves over nearly three octaves down over a span of 6 measures). The occurrence of these long lines might also be the result of the compositional framework and the wider harmonic space provided by the modal structure, which enables Chambers to play melodically rather than harmonically.

The B sections are tonally more ambiguous. In fact, Chambers often avoids playing a tone of the Eb minor triad on beat 1.36 He does so occasionally: 5 out of 16 measures during Davis’, 6 out of 16 during Coltrane’s, and 2 out of 16 during Adderley’s solo.37 The most frequent tone on the first beat over all B sections is A (Fig. 6). In most B sections, E is a rather rarely used pitch class. He plays D, A or B more frequently, suggesting D Ionian, A Mixolydian, or B Aeolian (Fig. 4 & 5). He also plays ambiguous or clear D or A triads, or tones taken from both triads, rather regularly.37 In fact, Chambers makes use of this destabilizing effect as early as the first B section of Davis’ solo. While he is not playing a clear D triad there, but something closer to an A triad, he starts emphasizing the D triad with the 2nd B section of Davis’ solo.

In conclusion, Chambers clearly outlines the D Dorian mode in the A sections with frequent long and wide lines that often imply virtual chord changes, mainly between the tonic and the minor dominant, albeit not in any systematic fashion. The (harmonic) tension implied by the B section is also reflected in the destabilizing treatment employed by Chambers, not only by avoiding Eb Dorian, but also by utilising frequently fast changes of implied chords and heightened chromaticism. Only 8% of the tones in sections A are chromatic, i.e. not included in the D Dorian mode, while 14% of the tones played in section B are

35 M. 14 (Davis’ solo), m. 14, m. 64 (Coltrane’s solo) and m. 1, m. 7, m. 61, m. 62 (Adderley’s solo).
36 He does so occasionally: 5 out of 16 measures during Davis’, 6 out of 16 during Coltrane’s, and 2 out of 16 during Adderley’s solo.
37 M. 50 and 52 (Davis’ solo), m. 19, 50, and 52 (Coltrane’s solo), m. 19 and 23 (Adderley’s solo).
chromatic tones within the context of E♭ Dorian. Moreover, Chambers’ lines every now and then imply modes other than the prescribed or neighbouring modes (e. g. E♭ Dorian at the end of A sections). Furthermore, he also uses bass ostinatos (riffs) prominently during the first A sections of Davis’ and Coltrane’s second choruses, which will become a future staple of modal jazz.38

Comparison with Coltrane’s »Impressions«

In 1961, Coltrane composed »Impressions«, which is based on the AABA scheme of »So What«. »Impressions« became a staple of his work during his middle or »modal« period (1959–1964), with frequently very long and extensive solos. It is interesting to compare the pitch class hierarchies of his solos from the two recordings of »Impressions« included in the Weimar Jazz Database. One »Impressions« solo (JC61) stems from 1961 (published on the album Impressions), with a staggering number of 32 choruses, and one from 1963, with nine choruses (JC63, published on Afro Blue Impressions). The pitch class rank plots separated for each of the both sections of Coltrane’s solos on »So What« and the two »Impressions« recordings are depicted in Fig. 7. The pitch class frequencies within the three solos are significantly different for each form part (Bootstrap $\chi^2$ test with 100 simulations, $BF_{0.001;100} = 98$ for A sections and $BF_{0.001;100} = 100$ for B sections). For the A sections, the fifth A is the most frequent pitch, likewise the fifth B♭ for the B sections of »Impressions« – in contrast to the B sections of »So What«. The most striking difference to »So What« is the emphasis on the third and sixth scale degrees (F and B) in the A sections of the »Impressions« solos, which are even more frequent than the tonic D, which ranks at only 5th (JC ’61) and 4th place (JC ’63). Furthermore, the three- or four-tier hierarchies observed for »So What« are less clearly discernible in both versions of »Impressions«. To corroborate this observation, we determined optimal cluster solutions of pitch class frequencies for all three solos of Coltrane and for each form part separately, using the method delineated above (Table 2). For each form part, a four cluster solution was found. The quality Q of the cluster solution, i.e. the ratio between cluster variance and total variance, is greater than 90% in all solutions, ranging from 99% for the B section of »So What« to a mere 92% for the B section of JC ’61. The difference between the centres of the highest and lowest cluster for both form parts is the smallest in JC ’61 (about 0.1). Likewise, the standard deviation of consecutive differences between clusters is rather small for both form parts. The pitch class frequencies of the B sections of JC ’61 exhibit no terraces at all and line up along a descending line. This could be interpreted as an intensification of the tonal instability in the B sections already observed in »So What«. However, this have to be examined further in detail, taking into account the sheer length of Coltrane’s ’61 solo.

38 See also Mike Downer, The jazz bass line book, Hechingen 2004, pp. 115–119.
39 The Bayes Factors here $BF_{α;N}$ is defined as the ratio of significant tests to anticipated random significances at level $α$ for N bootstrap samples.
Figure 7 Pitch classes ranked by frequency for all basic form parts for three modal solos by John Coltrane. JC = »So What«, JC '61 = »Impressions« 1961, JC '63 = »Impressions« 1963.

<table>
<thead>
<tr>
<th>Solo</th>
<th>Section</th>
<th>No. Clusters</th>
<th>Q</th>
<th>AM</th>
<th>SD</th>
<th>Range</th>
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<tr>
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<td>0.977</td>
<td>0.070</td>
<td>0.046</td>
<td>0.210</td>
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</tbody>
</table>

Table 2 Optimal cluster solutions for three modal solos by John Coltrane.
Note: JC = »So What«, JC '61 = »Impressions« (1961), JC '63 = »Impressions« (1963). Section = form section A or B, No. Clusters = number of optimal clusters, Q = between cluster variance/total variance, AM = arithmetic mean of differences between consecutive clusters centres, SD = standard deviation of differences between consecutive cluster centres, Range = difference between the centres of the first and last clusters.

Comparison with pitch classes over Dm7
To round out the picture, we investigated the pitch class hierarchies over Dm7 chords in 80 solos from the Weimar Jazz Database, ranging over six different styles (traditional jazz, swing, bebop, hardbop, cool jazz, and postbop). The pitch class rank plots are depicted in Fig. 8. In all styles, the third F is the most or second most important pitch class. Traditional and, to a lesser degree, swing players clearly outline the four chord pitch classes D, F, A, and C. The fifth A is de-emphasized in all styles except swing and postbop, where it is ranked in fourth place. Beginning with bebop, the fourth G is ranked very high: this might come from anticipating a G7 chord which is the most likely chord to follow in an ii7-V7 chord.
progression. The major sixth B is the least used pitch class in three styles, but more prominent in the older styles traditional and swing, again possibly due to a following G7 in a C major context. To further illuminate this fact, we also differentiated for pieces in C major (14) and pieces in other keys (Fig. 9), since there is a high probability for Dm7 performing the function of the ii7 in C major.

All in all, the comparison shows clear differences between improvisations over a Dm7 chord in a standard tonal context and in a modal context. In the modal context, the (supposed) pitch class hierarchy for the D Dorian mode is much more clearly carved out.

![Figure 8](image)

**Figure 8** Pitch classes ranked by frequency over Dm7 chords in 80 solos from the *Weimar Jazz Database*, differentiated according to style (traditional jazz, swing, bebop, hardbop, cool jazz, and postbop).

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40 Unfortunately, the *Weimar Jazz Database* does not contain harmonic analyses yet, so there is no information on the context of the Dm7 (whether it is followed by a G7 or functions as a tonic or iv). We must therefore rely on this approximation of context.
Conclusions

In this paper, we investigated pitch class hierarchies in modal jazz improvisation within a case study of the solos and bass lines of »So What«. We were able to show that the tones played by Davis, Coltrane, and Adderley, as well as by Chambers, could be described as pitch class hierarchies. In general, the tones played most frequently in the A sections are the tonic and fifth of D Dorian, followed by the second, fourth, third, and seventh scale degree. This usage clearly emphasizes the modal character of the piece, particularly in the A sections. The major sixth degree is used less often, but played more regularly than the remaining pitch classes outside the D Dorian scale, which constitute about 10% of all tones in the solos as well as in Chambers’ bass lines. In contrast to bebop and hardbop improvisation, the players use chromaticism only for colour and for smoothing out lines with passing tones.

However, the pitch class hierarchy is clearly destabilized within the B sections of »So What«. This destabilization reaches such an extent that the B sections can hardly be conceived as being played in E♭ Dorian. Although all players, including Chambers, mainly use the material of E♭ Dorian, the pitch class hierarchy is, however, rather blurred. In actual fact, the mode of the B section could be more accurately described as a mixture of D♭ Ionian, E♭ Dorian, A♭ Mixolydian, and sometimes B♭ Aeolian. Chambers’ bass lines tend towards D♭ Ionian, sometimes towards A♭ Mixolydian (e. g. B♭-I in Coltrane’s solo). This improvisational approach in the B section appears to be related to its tension generating function within the AABA chorus form.

On the whole, we were able to concretize the modal approach towards improvisation outlined rather vaguely by Davis and Evans in interviews (Davis) and the liner notes to Kind of Blue (Evans) with a close examination of the pitch class distributions and hierarchies in »So What«. Moreover, along with the mo-
tivic improvisation prevalent in Davis’ and Coltrane’s solos, many influential modal techniques are identifiable, e. g. brief outside phrases (»side-slippings«), or, in Chambers’ walking bass, long stepwise diatonic or chromatic lines, pedal points or bass ostinatos.

As a comparison with the pitch class hierarchies in two improvisations by Coltrane in »Impressions« from 1961 and 1963 has shown, Coltrane further developed his modal playing with an emphasis on the defining tones of Dorian, the minor third and the major sixth, while relaxing the overall pitch class hierarchy. However, he still uses the B section as a contrasting part with more destabilizing pitch classes.

The usage of the scale degrees of D Dorian in »So What« differs significantly from the usage of pitch classes while improvising over Dm7 chords in a range of jazz styles with functional harmonic organisation. There, the third F is the most frequently employed pitch class. Anticipations of pitch classes of a following G7 chord also occur.

Our detailed examinations were greatly facilitated by the use of digitally available music transcriptions as well as by computer-based automated analysis. While in traditional music analysis the researcher relies strongly on his or her listening experience – and therefore on his or her ability to find and remember significant features within music scores or recordings – computer-based and data-driven procedures allow for a fast and reliable detection of musical features such as pitch classes. This holds true in particular for the analysis of certain repertoires and music corpora, but also, as we have attempted to demonstrate, for the analysis of single and relatively short jazz improvisations. Moreover, further statistical analyses and visualisations of the results are facilitated by using software tools, and these could in turn enhance interpretation and lead to new insights. Of course, computational analyses are not restricted to pitches and pitch class hierarchies but can be applied to every musical feature. However, computer-based analysis procedures require clear, unambiguous terminology, with every musical feature explicitly defined – since a machine cannot compute using vague terms or calculation rules.

»So What« was a starting point for the development of a variety of practices of harmonic organisation and improvisation in postbop jazz. Our case study is, accordingly, just a first step in the analysis of a variety of practices featured in modal jazz improvisation. It might be continued with additional follow-up studies of other recordings and other musicians, taking both the soloists and their interactions with the bass lines and piano voicings into account. Some of the newly developed improvisational strategies of musicians such as Coltrane, David Liebman, or Michael Brecker have already been reflected in jazz theory and jazz pedagogy publications. However, the question as to exactly how and to what degree these theoretical and pedagogical approaches to modal improvisation actually characterize jazz musicians’ styles and improvisational practices has yet to be examined in detail, through an analysis of the recorded improvisations.

Appendix: Transcriptions

So What (Solo)  
Miles Davis
Walking Bass (Davis' Solo)

Paul Chambers

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CC BY-NC-ND 4.0 – © Schott Music GmbH & Co. KG
Walking Bass (Adderley's Solo)

Paul Chambers

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